

# Fundamental unit of area: black hole horizon statistical mechanics

Bernard F Schutz

May 20, 2009

# Horizon Generators

- ▶ Many suggestions that BH entropy comes from arrangements of 1-bit areas on horizon

# Horizon Generators

- ▶ Many suggestions that BH entropy comes from arrangements of 1-bit areas on horizon
- ▶ Holographic point of view reinforces this: horizon contains the data that determines exterior geometry

# Horizon Generators

- ▶ Many suggestions that BH entropy comes from arrangements of 1-bit areas on horizon
- ▶ Holographic point of view reinforces this: horizon contains the data that determines exterior geometry
- ▶ Make here a physical assumption: null generators of horizon are quantized, area quanta associated with them

# Horizon Generators

- ▶ Many suggestions that BH entropy comes from arrangements of 1-bit areas on horizon
- ▶ Holographic point of view reinforces this: horizon contains the data that determines exterior geometry
- ▶ Make here a physical assumption: null generators of horizon are quantized, area quanta associated with them
- ▶ They are “gravitons”, and the 1-bit datum is their spin helicity: up ( $\uparrow$ ) or down ( $\downarrow$ ).

# Horizon Generators

- ▶ Many suggestions that BH entropy comes from arrangements of 1-bit areas on horizon
- ▶ Holographic point of view reinforces this: horizon contains the data that determines exterior geometry
- ▶ Make here a physical assumption: null generators of horizon are quantized, area quanta associated with them
- ▶ They are “gravitons”, and the 1-bit datum is their spin helicity: up ( $\uparrow$ ) or down ( $\downarrow$ ).
- ▶ If there are  $N$  such gravitons and a fraction  $f$  are  $\uparrow$ ,  $1 - f$  are  $\downarrow$ , then one can make Schwarzschild ( $f = 1/2$ ) or Kerr ( $f \neq 1/2$ ).

# Horizon Generators

- ▶ Many suggestions that BH entropy comes from arrangements of 1-bit areas on horizon
- ▶ Holographic point of view reinforces this: horizon contains the data that determines exterior geometry
- ▶ Make here a physical assumption: null generators of horizon are quantized, area quanta associated with them
- ▶ They are “gravitons”, and the 1-bit datum is their spin helicity: up ( $\uparrow$ ) or down ( $\downarrow$ ).
- ▶ If there are  $N$  such gravitons and a fraction  $f$  are  $\uparrow$ ,  $1 - f$  are  $\downarrow$ , then one can make Schwarzschild ( $f = 1/2$ ) or Kerr ( $f \neq 1/2$ ).
- ▶ The entropy is the statistical entropy associated with rearrangements of identical gravitons.

# Entropy, Spin, and Mass

- ▶ Number of states:  $W = \frac{N!}{(fN)![(1-f)N]!}$



# Entropy, Spin, and Mass

- ▶ Number of states:  $W = \frac{N!}{(fN)![(1-f)N]!}$
- ▶ Entropy for large  $N$  (use Stirling's formula)

$$S = k \ln W = kN\alpha(f),$$

$$\alpha(f) := -f \ln f - (1-f) \ln(1-f) (> 0).$$

# Entropy, Spin, and Mass

- ▶ Number of states:  $W = \frac{N!}{(fN)![(1-f)N]!}$
- ▶ Entropy for large  $N$  (use Stirling's formula)

$$S = k \ln W = kN\alpha(f),$$

$$\alpha(f) := -f \ln f - (1-f) \ln(1-f) (> 0).$$

- ▶  $k\alpha(f)$  is the specific entropy, entropy per generator. For Schwarzschild,  $\alpha = \ln 2$ .

# Entropy, Spin, and Mass

- ▶ Number of states:  $W = \frac{N!}{(fN)![(1-f)N]!}$
- ▶ Entropy for large  $N$  (use Stirling's formula)

$$S = k \ln W = kN\alpha(f),$$

$$\alpha(f) := -f \ln f - (1-f) \ln(1-f) (> 0).$$

- ▶  $k\alpha(f)$  is the specific entropy, entropy per generator. For Schwarzschild,  $\alpha = \ln 2$ .
- ▶ Each graviton has spin  $\pm 2\hbar$ . Total spin  $J = 2(2f - 1)\hbar N$ .

# Entropy, Spin, and Mass

- ▶ Number of states:  $W = \frac{N!}{(fN)![(1-f)N]!}$
- ▶ Entropy for large  $N$  (use Stirling's formula)

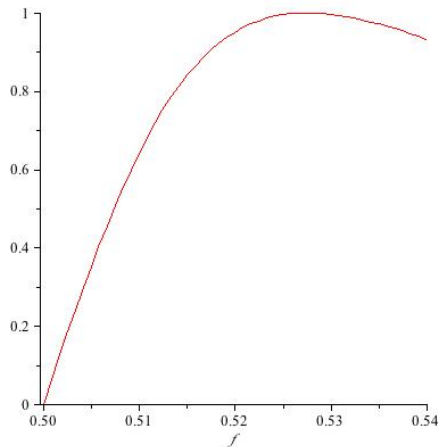
$$S = k \ln W = kN\alpha(f),$$

$$\alpha(f) := -f \ln f - (1-f) \ln(1-f) (> 0).$$

- ▶  $k\alpha(f)$  is the specific entropy, entropy per generator. For Schwarzschild,  $\alpha = \ln 2$ .
- ▶ Each graviton has spin  $\pm 2\hbar$ . Total spin  $J = 2(2f - 1)\hbar N$ .
- ▶ Bekenstein-Hawking entropy  $S = kA/4\hbar$ ,  
 $A = 8\pi[M^2 + \sqrt{(M^4 - J^2)}]$ , so that  $M^2 = A/16\pi + 4\pi J^2/A$ .

Result: Spin  $a/M$  of the hole

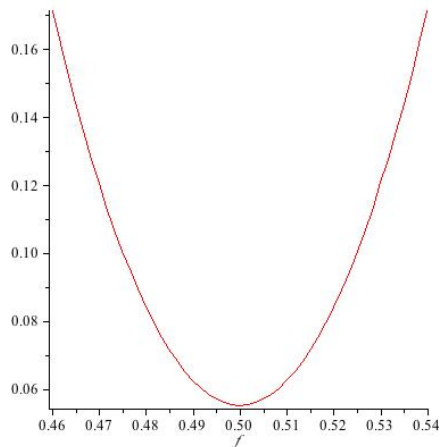
$$a/M = 2(2f - 1) \left[ \frac{\alpha(f)}{4\pi} + \frac{4\pi(2f - 1)^2}{\alpha(f)} \right]^{-1}.$$



## Result: Mass of the hole

$$M^2 = \hbar N \left[ \frac{\alpha(f)}{4\pi} + \frac{4\pi(2f-1)^2}{\alpha(f)} \right].$$

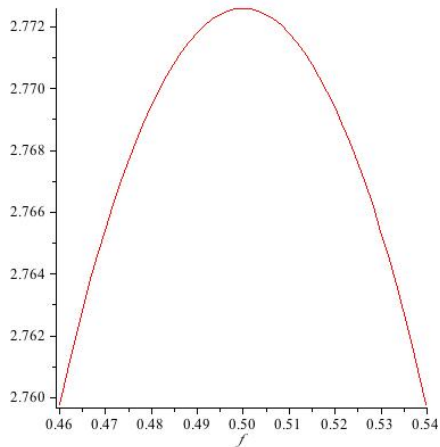
$M^2/\hbar N$ :



## Result: Fundamental Quantum of Area

$$\delta A = A/N = 4\hbar\alpha(f)$$

Only tiny variation between Schwarzschild and Kerr:  $\delta A/\hbar$



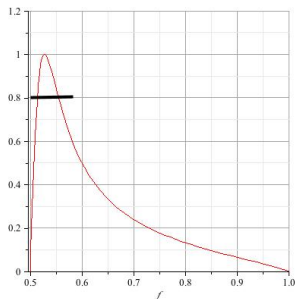
## Energy per graviton

$$E_g = M/N = \frac{\hbar}{M} \left[ \frac{\alpha(f)}{4\pi} + \frac{4\pi(2f-1)^2}{\alpha(f)} \right].$$

Sensible: typical wavelength of order  $M$ .

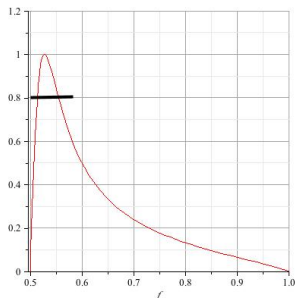


# Two ways of making a Kerr Hole?



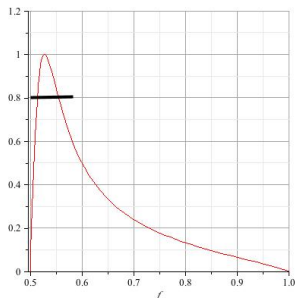
- ▶ Two different values of  $f$  for each  $a/M$ . Larger  $|f|$  seems not physical. Why?

# Two ways of making a Kerr Hole?



- ▶ Two different values of  $f$  for each  $a/M$ . Larger  $|f|$  seems not physical. Why?
- ▶ For fixed  $N$ , larger  $f$  means larger  $M$  and smaller entropy. Unstable?

# Two ways of making a Kerr Hole?



- ▶ Two different values of  $f$  for each  $a/M$ . Larger  $|f|$  seems not physical. Why?
- ▶ For fixed  $N$ , larger  $f$  means larger  $M$  and smaller entropy. Unstable?
- ▶ Why does temperature vanish at  $a/M = 1$ ?

# Holographic Noise Workshop: Day 1 Summary

- ▶ Talks by Craig, Bruce, Bernard, Klaus
- ▶ ,4- $\dot{\iota}$  Very challenging counter-example by Gerhard, not fully understood.
- ▶ Conclusion: may not be able to understand this without an experiment!

# Holographic Noise Workshop: Day 1 Summary

- ▶ Talks by Craig, Bruce, Bernard, Klaus
- ▶ Learned some things: holographic effect is NOT quantum gravity
- ▶ ,4- $\zeta$  Very challenging counter-example by Gerhard, not fully understood.
- ▶ Conclusion: may not be able to understand this without an experiment!

# Holographic Noise Workshop: Day 1 Summary

- ▶ Talks by Craig, Bruce, Bernard, Klaus
  - ▶ Learned some things: holographic effect is NOT quantum gravity
  - ▶ Big questions not fully clarified: correlations between  $x$  and  $y$ ? Entanglement?
  - ▶ ,4- $\dot{\iota}$  Very challenging counter-example by Gerhard, not fully understood.
- 
- ▶ Conclusion: may not be able to understand this without an experiment!

# Holographic Noise Workshop: Day 1 Summary

- ▶ Talks by Craig, Bruce, Bernard, Klaus
  - ▶ Learned some things: holographic effect is NOT quantum gravity
  - ▶ Big questions not fully clarified: correlations between  $x$  and  $y$ ? Entanglement?
  - ▶ ,4- $\dot{\iota}$  Very challenging counter-example by Gerhard, not fully understood.
- 
- ▶ Conclusion: may not be able to understand this without an experiment!

# Holographic Noise Workshop: Day 1 Summary

- ▶ Talks by Craig, Bruce, Bernard, Klaus
- ▶ Learned some things: holographic effect is NOT quantum gravity
- ▶ Big questions not fully clarified: correlations between  $x$  and  $y$ ? Entanglement?
- ▶ ,4- $\dot{\iota}$  Very challenging counter-example by Gerhard, not fully understood.
- ▶ On theory side, lacking a mathematical description in language of quantum mechanics: what operators have commutation relations, what are the observables?
- ▶ Conclusion: may not be able to understand this without an experiment!



# Holographic Noise Workshop: Day 1 Summary

- ▶ Talks by Craig, Bruce, Bernard, Klaus
- ▶ Learned some things: holographic effect is NOT quantum gravity
- ▶ Big questions not fully clarified: correlations between  $x$  and  $y$ ? Entanglement?
- ▶ ,4- $\dot{c}$  Very challenging counter-example by Gerhard, not fully understood.
- ▶ On theory side, lacking a mathematical description in language of quantum mechanics: what operators have commutation relations, what are the observables?
- ▶ Conclusion: may not be able to understand this without an experiment!